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M-theory lift of brane-antibrane systems and localised closed string tachyons

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Abstract: We discuss the lift of certain D6-$\overline{\text{D6}}$-brane systems to M-theory. These are purely gravitational configurations with a bolt singularity. When reduced along a trivial circle, and for large bolt radius, the bolt is related to a non-supersymmetric orbifold type of singularity where some closed string tachyons are expected in the twisted sectors. This is a kind of open-closed string duality that relates open string tachyons on one side and localised tachyons in the other. We consider the evolution of the system of branes from the M-theory point of view. This evolution gives rise to a brane-antibrane annihilation on the brane side. On the gravity side, the evolution is related to a reduction of the order of the orbifold and to a contraction of the bolt to a nut or flat space if the system has non-vanishing or vanishing charge, respectively. We also consider the inverse process of reducing a non-supersymmetric orbifold to a D6-brane system. For $\mathbb{C}^2/\mathbb{Z}_N \times \mathbb{Z}_M$, the reduced system is a fractional D6-brane at an orbifold singularity $\mathbb{C}/\mathbb{Z}_M$.

Keywords: M-Theory, D-branes, String Duality, Tachyon Condensation.

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Contents

1. Introduction

It is well known that the lift to M-theory of a system of parallel D6-branes [1, 2] corresponds to a purely geometric background, the Taub-NUT metric. When the position of N of these D6-branes coincide, one gets an A_{N-1} singularity at a point in the multi Taub-NUT space. In this paper, we would like to make a step forward in the relation between the physics of D6-branes at strong coupling and purely gravitational backgrounds in eleven dimensional supergravity by studying the lift of a system of coincident D6-$\overline{\text{D6}}$ branes to M-theory. We shall primarily be concerned with the geometry describing these configurations, its evolution as branes and antibranes annihilate each other[1] and some similarities between the qualitative patterns that we find in this evolution and some recent results on the evolution due to the condensation of localised closed string tachyons in non-supersymmetric orbifold singularities [4]–[9].

In particular, we shall study the lift to M-theory of the generically non-BPS configurations found in [10, 11, 12] preserving ISO$(1,6) \times$ SO(3). The latter depend on three parameters. The subset of configurations in which we will be interested in corresponds to setting one of them to zero. These particular geometrical configurations look like $\mathbb{R}^{1,6} \times \mathcal{M}_4$, for some curved four dimensional manifold. It turns out that $\mathcal{M}_4$ has a *bolt* type singularity, that is, a locus of conical singularities, whose conical defects depend on the mass and the

[1] See, for instance, [3].

charge of the configuration. The brane-antibrane annihilation expected in the open string description gives rise to a reduction in the size of the bolt and a desingularization of the conical singularities, by which they become "less conical". In the sector of non-vanishing charge, the bolt becomes a nut, whereas in the vanishing charge sector, the bolt disappears.

Locally, when the size of the bolt is big, the system looks like $\mathcal{M}_4 \sim \mathbb{C} \times \mathbb{C}/\mathbb{Z}_M$. The size of the bolt is proportional to the product of the number of branes, the number of antibranes and $(g_s l_s)^2$. Thus, big bolt limit means that $g_s^2 N\bar{N}$ is big, i.e. the number of branes and antibranes should be large in order to keep a small string coupling. Thus, by reducing along a trivial circle, the original D6-$\overline{\text{D6}}$ system is related to a $\mathbb{C}/\mathbb{Z}_M$ orbifold in the forementioned limit. Whenever $M \neq 1$, there are closed string tachyons in the twisted sectors. Recent studies [5, 6, 7] suggest that this system evolves to flat space making the cone "less conical" by a sequence of transitions

$$\mathbb{C}/\mathbb{Z}_{2l+1} \to \mathbb{C}/\mathbb{Z}_{2l-1} \ldots \to \mathbb{C}/\mathbb{Z}_{2l'-1} \quad (l' < l)\,.$$

Our qualitative comparison in the large bolt limit suggests a relation between brane-antibrane annihilation and twisted tachyon evolution. And in particular, each transition $(l \to l-1)$, which reduces the order of the orbifold by two, is related to the annihilation of a D6-$\overline{\text{D6}}$ pair.

In the second part of the paper, and motivated by the previous relation, we start from a non-supersymmetric orbifold acting on $\mathbb{C}^2$ in type IIA, lift the configuration to M-theory using a trivial transverse circle and reduce it along a non-trivial circle in $\mathbb{C}^2$. One expects such system to be the local description for an unstable system of branes. In particular, we consider $\mathbb{C}^2/\mathbb{Z}_N \times \mathbb{Z}_M$, where each abelian group preserves different supersymmetry, so that the full orbifold is non-supersymmetric. The interpretation of the reduced system is in terms of fractional D6-branes living on a $\mathbb{C}/\mathbb{Z}_M$ singularity. Here the closed string tachyonic instabilities cannot be mapped to open string tachyons as in the previous case.

The organisation of the paper is as follows. In section 2, we revisit the construction of supergravity solutions given in [10, 11], paying attention to the particular case of D6-$\overline{\text{D6}}$-branes. These solutions depend on three parameters. We discuss the scaling limits leading to BPS configurations, generalising the discussion in [11]. We consider the lift of such configurations to M-theory and argue why it is interesting for us to set one of the parameters to zero. In this way, we get a two parameter family of solutions, where the parameters can be mapped to the Ramond-Ramond (RR) charge and mass of the system. In section 3, we analyse this solution in detail, both in the charged and uncharged sectors. In section 4, we discuss the evolution of the system and we compare the open and closed string descriptions. section 5 is devoted to the study of the inverse problem: going from a non-supersymmetric orbifold to a local description of a system of D6-branes. In particular, we consider a $\mathbb{C}^2/\mathbb{Z}_N \times \mathbb{Z}_M$ non-supersymmetric orbifold.

2. From brane-antibranes to M-theory

In [10], the most general solution to the supergravity equations of motion with $\mathrm{ISO}(1,p) \times \mathrm{SO}(9-p)$ symmetry and carrying the appropriate Ramond-Ramond (RR) charge was integrated. It was subsequently interpreted in [11] as a system of coincident Dp-$\overline{\mathrm{D}p}$ branes. In this work, we shall concentrate on the D6-$\overline{\mathrm{D6}}$ system. In the Einstein frame, the configuration is described by

$$\begin{aligned}
g_E &= e^{2A(r)} ds^2(\mathbb{E}^{1,6}) + e^{2B(r)}(dr^2 + r^2 d\Omega_2^2) \\
\Phi &= \Phi(r) \\
C_{(7)} &= e^{\Lambda(r)}\, \mathrm{dvol}(\mathbb{E}^{1,6})\,,
\end{aligned} \tag{2.1}$$

where g_E is the ten dimensional metric, Φ is the dilaton and $C_{(7)}$ is the RR seven form potential. The set of scalar functions characterising the above configuration is given by

$$\begin{aligned}
A(r) &= -\frac{3}{64} c_1\, h(r) - \frac{1}{16}\log[\cosh(kh(r)) - c_2 \sinh(kh(r))] \\
B(r) &= \log[f_-(r) f_+(r)] - 7A(r) \\
\Phi(r) &= c_1\, h(r) + 12\, A(r) \\
e^{\Lambda(r)} &= -\sqrt{c_2^2 - 1}\frac{\sinh(kh(r))}{\cosh(kh(r)) - c_2 \sinh(kh(r))}\,,
\end{aligned} \tag{2.2}$$

where

$$\begin{aligned}
f_\pm &= 1 \pm \frac{r_0}{r} \\
h(r) &= \log\left[\frac{f_-(r)}{f_+(r)}\right] \\
k &= \sqrt{4 - \frac{7}{16} c_1^2}\,,
\end{aligned}$$

Thus, it depends on two dimensionless parameters $\{c_1\,, c_2\}$ defined in the ranges $c_2 \geq 1$, $-\frac{8}{\sqrt{7}} \leq c_1 \leq 0$, and a third one r_0, with dimensions of length satisfying $r_0 \geq 0$.

The charge (Q) and mass (M) of this solution were computed in [11] and we shall follow their conventions. They are expressed in terms of $\{r_0\,, c_1\,, c_2\}$ as follows

$$Q = 2\,\mathrm{P} \cdot k\, r_0 \sqrt{c_2^2 - 1} \tag{2.3}$$

$$M = \mathrm{P} \cdot r_0 \left[2c_2 \cdot k - \frac{3}{2} c_1\right], \tag{2.4}$$

where $\mathrm{P} = \frac{1}{16}\frac{V_6}{G_{10}^N}$, V_6 being the spacelike volume spanned by the branes and G_{10}^N stands for the ten dimensional Newton's constant. Written in string units, $\mathrm{P} = \frac{\pi}{(2\pi)^7}\frac{V_6}{g_s^2 l_s^8}$, where g_s is the string coupling constant and l_s is the string length $l_s^2 = \alpha'$.

Notice that in general the configuration is non-BPS ($M \neq Q$), as expected, and it is useful to introduce the difference between these observables

$$\delta M \equiv M - Q = \mathrm{P}\, r_0 \left[2k\left(c_2 - \sqrt{c_2^2 - 1}\right) - \frac{3}{2} c_1\right]. \tag{2.5}$$

2.1 BPS limits

The first natural question to address is how to recover the well-known BPS configurations corresponding to N D6-branes (or $\overline{\rm D}$6-branes) from the general solution (2.1). At this point, we would like to point out that there are more possibilities than the one discussed in [11]. Indeed, the idea there was to take a certain scaling limit in the set of parameters $\{r_0\,, c_1\,, c_2\}$, or equivalently in $\{r_0\,, k\,, c_2\}$, such that the charge Q remains finite while $\delta M \to 0$. As discussed in [11], one possibility is to consider

$$r_0 \to \epsilon^{1/2}\, r_0\,, \qquad k \to \epsilon^{1/2} k\,, \qquad c_2 \to \frac{c_2}{\epsilon} \qquad \epsilon \to 0 \tag{2.6}$$

which can also be formulated in terms of c_1, by $c_1 \to -\frac{8}{\sqrt{7}} + \epsilon\frac{k^2}{\sqrt{7}}$.

The previous scaling limit is certainly not the only possibility, and as it will turn out important for us later on, we discuss a second possibility. Consider the following double scaling limit

$$r_0 \to \epsilon\, r_0\,, \qquad c_2 \to \frac{c_2}{\epsilon}\,, \qquad \epsilon \to 0 \qquad \left[c_1 \neq -\frac{8}{\sqrt{7}}\ \text{fixed}\right]. \tag{2.7}$$

It is clear that the charge (2.3) remains finite in the limit (2.7) and that δM vanishes, as required. As a further check, it is straightforward to analyse (2.2) in the above limit to get back the BPS metric [13] from (2.1).

2.2 M-theory lift

By rescaling the Einstein metric to the string frame and using the standard Kaluza-Klein ansatz, one derives a family of purely geometrical configurations in eleven dimensions described by the metric

$$\begin{aligned} g = {} & \left(\frac{f_-(r)}{f_+(r)}\right)^{-c_1/6} ds^2(\mathbb{E}^{1,6}) + \\ & + \left(\frac{f_-(r)}{f_+(r)}\right)^{7c_1/12} (f_-(r) f_+(r))^2 \left[\cosh(kh(r)) - c_2 \sinh(kh(r))\right] \left(dr^2 + r^2 d\Omega_2^2\right) + \\ & + \left(\frac{f_-(r)}{f_+(r)}\right)^{7c_1/12} \left[\cosh(kh(r)) - c_2 \sinh(kh(r))\right]^{-1} (dz + C_1)^2\,, \end{aligned} \tag{2.8}$$

where z stands for the spacelike coordinate along the M-theory circle with length at infinity $2\pi g_s l_s$ and $C_{(1)}$ is the magnetic dual one form to the previous RR 7-form $[dC_{(1)} = \star_{10} dC_{(7)}]$.

Notice that whenever $c_1 \neq 0$, the eleven dimensional geometry is not that of seven dimensional Minkowski spacetime times some curved manifold, but contains a warped factor. In the limit $r_0 \to 0$ keeping $c_1\,, c_2$ fixed, the geometry asymptotes to the maximally supersymmetric Minkowski spacetime.

One non-trivial check [11] for the above family of solutions (2.8) concerns the zero charge sector ($Q = 0$). Indeed, it has been known for a while the embedding in eleven dimensions [17] of the Kaluza-Klein dipole solution [18] describing a monopole-antimonopole

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pair separated by some distance. Studying such a solution in the limit of vanishing dipole size, one gets the configuration

$$g = ds^2(\mathbb{E}^{1,6}) + r^2\left(\Delta^{-1}(r)dr^2 + d\Omega_2^2\right) + \Delta(r)r^{-2}dx^2\,, \tag{2.9}$$

where the scalar function $\Delta(r)$ is defined by $\Delta(r) = r(r-2M)$, M being some constant parameter.

It is clear that the matching between (2.8) and (2.9) requires setting

$$c_1 = 0\,, \qquad c_2 = 1$$

to ensure the vanishing of the warped factor and charge, respectively. The same reasoning applies for a system of more than two monopoles. If we want the solution to remain as a seven dimensional Minkowski spacetime times some four dimensional manifold where the monopoles are living, one needs $c_1 = 0$. In this subspace , (2.8) becomes

$$g = ds^2(\mathbb{E}^{1,6}) + \left(1+\frac{r_0}{r}\right)^4(dr^2 + r^2 d\Omega_2^2) + \left(\frac{r-r_0}{r+r_0}\right)^2 dx^2\,. \tag{2.10}$$

Notice that (2.9) and (2.10) are equivalent, as expected, under the coordinate transformation:

$$r = \hat{r}\,(f_+(\hat{r}))^2\,,$$

where $\hat{r}$ stands for the radial coordinate in (2.10), provided the two constant parameters are identified as

$$M = 2\hat{r}_0\,.$$

Notice that the right hand side of the above coordinate transformation is invariant under the transformation $r_0/\hat{r} \to \hat{r}/r_0$. We shall see later that this symmetry is not restricted to the vanishing charge sector ($c_2 = 1$), but generalizes to $Q \neq 0$.

2.3 Two parameter solution in M-theory

In the following, we shall concentrate on the $c_1 = 0\,[k=2]$ subspace of solutions [12]

$$g = ds^2(\mathbb{E}^{1,6}) + \left[\frac{1-c}{2}f_-^4 + \frac{1+c}{2}f_+^4\right](dr^2 + r^2 d\Omega_2^2) + \frac{(f_+f_-)^2}{\left[\frac{1-c}{2}f_-^4 + \frac{1+c}{2}f_+^4\right]}(dz + C_{(1)})^2 \tag{2.11}$$

which includes (2.10) in the sector of zero charge [$c_2 \equiv c = 1$]. We would like to emphasise that such a subspace of configurations includes both the BPS ones, through the scaling limit (2.7), and the zero distance monopole-antimonopole pair solution (2.9). Since it contains a seven dimensional Minkowski spacetime, it allows us to concentrate on the physics of the four dimensional curved manifold, which is rather natural if one is interested in relating the physics of D6-$\overline{\text{D6}}$ at strong coupling with tachyon condensation in orbifold models in $\mathbb{C}^2$, whose local description close to the fixed point (singularity) consists of such a seven dimensional Minkowski spacetime times some four dimensional manifold.

The two parameters $\{r_0\,, c\}$ appearing in (2.11) can be mapped to the charge (2.3) and mass (2.4) of the system, which satisfy the quadratic relation:

$$M^2 = Q^2 + (4\mathrm{P}\cdot r_0)^2\,, \tag{2.12}$$

showing that the mass is bigger or equal to the charge. These parameters can be expressed in a much more physical way in terms of the number of branes (N) and anti-branes ($\overline{N}$) as

$$\begin{aligned} N - \bar{N} &= \frac{8}{g_s l_s} r_0 \sqrt{c^2 - 1} \\ N + \bar{N} &= \frac{8}{g_s l_s} r_0 c \,, \end{aligned} \tag{2.13}$$

or equivalently, by

$$\begin{aligned} r_0^2 &= \frac{(g_s l_s)^2}{16} N \bar{N} \\ c &= \frac{N + \bar{N}}{2\sqrt{N \bar{N}}} \,, \end{aligned} \tag{2.14}$$

As we can see from these formulae the radius of the bolt and the value of c are discrete, as only an integer number of branes is allowed.

Notice that the measure for the non-BPS character of the configuration (2.5) is proportional to the ratio

$$\delta M \propto V_6 \cdot \frac{R_s \cdot r_0}{l_p^9} \,, \tag{2.15}$$

where R_s is the radius of the M-theory circle and l_p is the eleven dimensional Planck length. A natural way of measuring the non-BPS character of the configuration in terms of D6-branes data is by the quotient

$$\frac{N + \bar{N}}{N - \bar{N}} = \frac{c}{\sqrt{c^2 - 1}} \,. \tag{2.16}$$

If there are only branes or antibranes, the quotient equals ± 1, which can only be achieved if $c \to \infty$. Notice that to keep the charge (2.3) fixed in that limit, one must take at the same time $r_0 \to 0$, which matches our discussion on BPS limits, in particular the scaling limit (2.7).

As we shall discuss more extensively in the next section, there is a *bolt* type singularity at r_0, both in the charged and non-charged sectors, for non-zero values of r_0. When approaching the supersymmetric configuration, the fate of the *bolt* singularity depends on the sector in which we are:

(i) If $Q \neq 0$, it gives rise to the usual *nut* singularity at $r = 0$ where the monopoles (or antimonopoles) are sitting. This is the source for the naked singularity of the D6-branes (or $\overline{\text{D6}}$-branes) at the origin [15].

(ii) If $Q = 0$, it gives rise to flat space.

3. Geometry of the solution

Let us analyse the geometry of solution (2.11). First of all, it is exactly the Taub-bolt singularity without imposing the absence of conical singularities [14, 12]. That can be seen explicitly by the change of radial coordinate [12]:

$$r' = \frac{1}{2}\left(r - m + \sqrt{r^2 - 2m\,r + l^2}\right),$$

and identifying the parameters in both solutions as $c = m/\sqrt{m^2 - l^2}$ and $r_0 = \sqrt{m^2 - l^2}/2$.

If we keep the charge fixed and take $r_0 \to 0$, or equivalently, we take the double scaling limit (2.7), we end up with the Taub-NUT metric:

$$g_4 = H(r)(dr^2 + r^2 d\Omega_2^2) + H(r)^{-1}(dz + C_{(1)})^2\,, \tag{3.1}$$

where $H(r) = 1 + 4r_0 c/r$, as expected for the BPS configuration ($M = Q$). In the limit close to the origin, the metric (3.1) reproduces the singularity of a $\mathbb{Z}_N$ orbifold (A_{N-1} singularity), where N is the number of branes defined previously, i.e. in D-brane units $H(r) = 1 + \frac{1}{2} g_s l_s N/r$. Indeed, close to the singularity located at $r = 0$, one can make the coordinate transformation

$$\hat{r} = 2\left(\frac{1}{2} g_s l_s N \cdot r\right)^{1/2},$$

which allows us to write the metric as

$$g_4 = d\hat{r}^2 + \frac{\hat{r}^2}{4}\left[d\theta^2 + (\sin\theta)^2 d\varphi^2 + \left(\frac{2dz}{l_s g N} + (1 - \cos\theta)d\varphi\right)^2\right]. \tag{3.2}$$

Taking into account that z has a period of $2\pi g_s l_s$ one gets that the circle parametrised by z has a conical behaviour like a $\mathbb{Z}_N$ orbifold.

The solution (2.11) is defined for $r \geq r_0$, the interior of the sphere $r = r_0$ not belonging to the solution. However, it is interesting to point out the existence of an isometry, the in&out symmetry, that relates $r \ll r_0$ with $r \gg r_0$,

$$\frac{r_0}{r} \to \frac{r}{r_0}\,,$$

The geometry far away from $r \sim r_0$ has the same assymptotic behaviour as in the supersymmetric configuration. Thus, any source of instability reflected in the geometry has to be in the region $r \sim r_0$, at which we shall now look in detail.

Let us start our analysis in the charged sector ($Q \neq 0$). Whenever the configuration is non-BPS, the metric has a *bolt* singularity at $r = r_0$. The bolt is a sphere of radius proportional to r_0 with conical singularities on it. To study these singularities, we can examine the metric (2.11) close to the bolt, by introducing the distance to the bolt as a coordinate ($y = r - r_0$) and concentrating on the region $y \ll r_0$. After a trivial rescaling of the new radial coordinate, the four dimensional metric looks like

$$g_4 = dy^2 + 8(1+c)r_0^2 d\Omega_2^2 + \frac{y^2}{16(1+c)^2 r_0^2}(dz + C_{(1)})^2\,, \tag{3.3}$$

Thus, close to the bolt, the periodicity of the compact coordinate $x = z/l_s$ is reduced by a factor

$$\frac{1}{L} \equiv \frac{l_s g_s}{4(1+c)r_0} = \frac{2}{N + \bar{N} + 2\sqrt{N\bar{N}}}\,, \tag{3.4}$$

which indeed points out to the existence of conical singularities whose angular deficit is $2\pi\frac{L-1}{L}$. Notice that these singularities are located on a sphere of radius $\mathrm{R}_{\mathrm{bolt}} = 2\sqrt{2}\sqrt{1+c}\,r_0$, whose area is

$$A = 32\pi(1+c)r_0^2 = \pi(l_s g_s)^2(N + \bar{N} + 2\sqrt{N\bar{N}})\sqrt{N\bar{N}}\,. \tag{3.5}$$

Notice that the area takes discrete values depending on the integer numbers representing the number of branes and antibranes.

Even though the scalar curvature vanishes on the bolt, due to the existence of the conical singularities, one might wonder about higher order corrections to the eleven dimensional effective action close to the bolt. To clarify this issue, one can analyse the behaviour of the square of the Riemann tensor. Such corrections would be suppressed whenever

$$l_p^4\, R_{MNPR}R^{MNPR} \ll 1\,.$$

Working in the regime in which the number of branes is of the same order as the number of antibranes ($N \sim \bar{N}$), the above constraint looks like

$$l_p^4\, R_{MNPR}R^{MNPR} \sim \left(\frac{l_p}{r_0}\right)^4 \sim \left(g_s^{2/3}\cdot N\right)^{-4} \ll 1\,,$$

Therefore such corrections can be neglected when the size of the bolt is big in eleven dimensional Planck units, or equivalently

$$g_s^{2/3}\cdot N \gg 1\,, \qquad N \sim \bar{N}\,. \tag{3.6}$$

Notice that in order to keep the string coupling constant small, the number of branes must be large. This is the approximation we would like to use.

When the size of the bolt is big ($r_0/l_p \gg 1$), the metric (3.3) close to the bolt is a huge sphere times a cone. Furthermore, in the regime (3.6), the effect of $C_{(1)}$ is negligible.[2] Thus, locally, the four dimensional manifold $\mathcal{M}_4$ looks like

$$\mathcal{M}_4 \sim \mathbb{C}\times\mathbb{C}/\mathbb{Z}_L\,.$$

That such a description allows an orbifold singularity $\mathbb{C}/\mathbb{Z}_L$ interpretation can be further checked by using (3.4) in the regime (3.6), which ensures that L is an integer number.

These orbifold singularities have always closed string tachyons in the twisted sectors. In the next section, we shall compare the annihilation of brane-antibrane pairs expected in the open string description, with the sequences of transitions for $\mathbb{C}/\mathbb{Z}_{2l+1}$ orbifolds discussed in [5], and we shall see that they are qualitatively the same.

[2] Globally the structure of the space can be undertood as a $\mathbb{Z}_L$ vector bundle over a trivial $\mathbb{Z}_L$-space S^2. That means that the $\mathbb{Z}_L$ is acting trivially on the sphere while rotating the fibre $\mathbb{C}$. The charge Q of the system specifies the first Chern number as in the supersymmetric case.

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3.1 Same number of branes and antibranes

We shall now move to the vanishing charge sector, that is, the one with the same number of branes and antibranes, i.e. $N = \bar{N}$. In this case, the metric reduces to (2.10) and depends on a single parameter r_0, which can be written in terms of the number N of D6-$\overline{\mathrm{D6}}$ pairs as

$$r_0 = \frac{1}{4} g_s l_s \cdot N \,.$$

Since $C_{(1)}$ vanishes, the surfaces r = constant are trivial fibrations $S^1 \times S^2$. The assymptotic geometries are $\mathbb{R}^{1,9} \times S^1$, whereas close to $r \sim r_0$, one can check, proceeding in an analogous way to the previous discussion, that the bolt structure remains. In this case, the deficit in the periodicity is $1/2N$. That means that for an integer number of D6-branes the system has an orbifold interpretation as a $\mathbb{Z}_{2N}$ orbifold.

The scalar curvature vanishes everywhere, as it corresponds to a solution of Einstein supergravity equations of motion with no matter, whereas the squared of the Riemann tensor is given by

$$R_{MNPR} R^{MNPR} = 192 \frac{r^6 r_0^2}{(r + r_0)^{12}} \,,$$

which has a maximum at $r = r_0$. Once more, the gravity approximation is reliable in the large bolt limit.

4. Evolution of the system

When one trivially reduces the previous M-theory configurations (2.11) by adding an extra transverse compact circle, one finds a generically non supersymmetric purely gravitational (geometrical) type-IIA configuration. Thus, the analysis of singularities discussed above still applies to this geometry.

We are thus left with two different descriptions in type IIA of a single M-theory configuration: first, the brane-antibrane system and on the other hand, geometrical configurations with conical singularities located on a sphere. Furthermore, in the limit of big bolt (3.6), the geometry of the conical singularities is locally given by that of an orbifold type, $\mathbb{C} \times \mathbb{C}/\mathbb{Z}_N$. Thus, it is clear that both systems contain tachyons; the brane-antibrane system in the open string sector from strings stretching between a brane and antibrane, whereas in the orbifold side, there are closed string tachyons in the twisted sectors. These tachyons can be understood as localised on the bolt. Some properties of this kind of closed string twisted sectors and their possible evolution have been analysed in [5, 6, 7, 8]. In the following, we shall show that the expected annihilation of brane-antibrane pairs in the open string side matches the reduction in the order of the non-supersymmetric orbifold observed in the previous cited references.

We can consider the evolution of the system in the (M, Q) parameter space. In the D6-brane picture, we expect branes to annihilate the antibranes so that the total charge is preserved. The mass will decrease up to a supersymmetric system, $M = Q$, in which we are left either with all branes or all antibranes. This process is expected to be a discontinuous

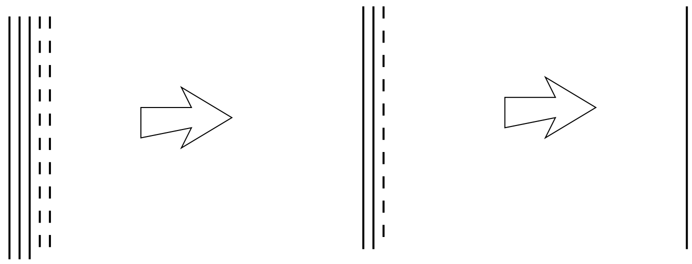

Figure 1: Schematic representation of the D6 anti-D6-brane annihilation to the supersymmetric configuration.

process: branes and antibranes are annihilated in pairs as closed string fields will be emitted to the bulk. We expect a sequence[3]

$$(M, Q) \to (M-2, Q) \to (M-4, Q) \ldots \to (Q, Q)$$

This process is represented schematically in figure 1.

When considered from the M-theory effective description in terms of a classical solution of the supergravity equations of motion, the latter depends on two continuous parameters: M and Q. Nevertheless, one can study the evolution in the geometry of the family of configurations by moving in such a two dimensional parameter space. Indeed, we are interested in studying the decrease in the mass M while keeping the charge Q fixed. It is clear that such a motion requires a decrease of r_0 while c increases "along the flow". Heuristically, we can think of $M = N_1 + 2\bar{N}$ and $Q = N_1$ as the starting point of the flow. The value of r_0 is thus determined to be

$$r_0 = \frac{g_s l_s}{4} \sqrt{\bar{N}^2 + N_1 \cdot \bar{N}}\,,$$

The motion along the flow we are interested in, is described by decreasing the parameter $\bar{N} \to \bar{N} - 2$, simulating the annihilation of a brane and antibrane. One can formally take the limit $\bar{N} \to 0$ and get the BPS configuration as expected. In the r_0, c parameter space this flow can be seen as a curve going to $r_0 \to 0$ and $c \to \infty$ (see figure 2).

This flow has two effects: the radius of the bolt goes to zero and the conical singularity gets 'less' conical with a factor $1/(M + \sqrt{M^2 - Q^2})$. When the system arrives at the supersymmetric configuration, the bolt disappears into a nut and a supersymmetric orbifold singularity remains at the origin $\mathbb{C}^2/\mathbb{Z}_Q$. See figure 3.

One very interesting case is when the number of branes N is exactly the same as the number of antibranes $\bar{N}$. In this case, the flow corresponds to a straight line at $c = 1$, and the decrease in r_0 is directly related to the decrease in $M = 2N = r_0$. Then close to the bolt there is an orbifold description as $\mathbb{C}/\mathbb{Z}_{2N}$. The process of annihilating branes and antibranes takes $N \to N - 2$. From the orbifold point of view that corresponds to a transition $\mathbb{C}/\mathbb{Z}_{2N} \to \mathbb{C}/\mathbb{Z}_{2(N-1)}$. Notice that this process is very similar to the one found by [5] where the orbifold singularity is desingularising till reaches the flat space by

[3] We call now M the number of branes plus antibranes, and Q its difference.

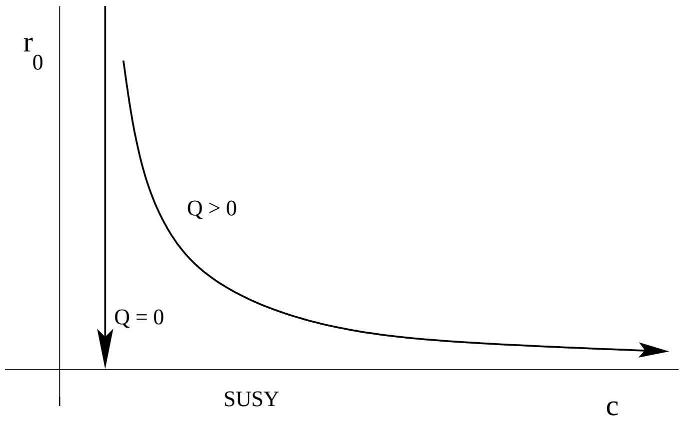

Figure 2: Flow in the r_0 and c parameter space representing the annihilation of brane antibrane-pairs.

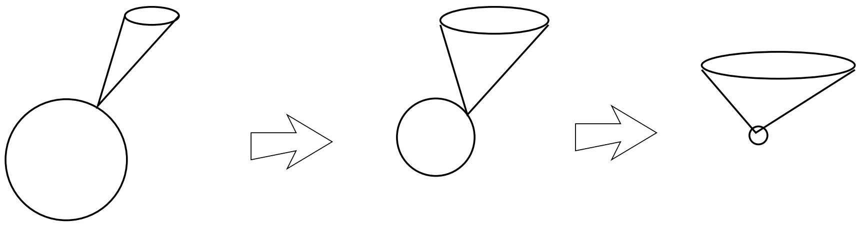

Figure 3: M-theory lift of the brane anti-brane annihilation process. The two effects are the reduction of the bolt to a point and the expanding of the cone to get a supersymmetric singularity $\mathbb{C}^2/\mathbb{Z}_N$.

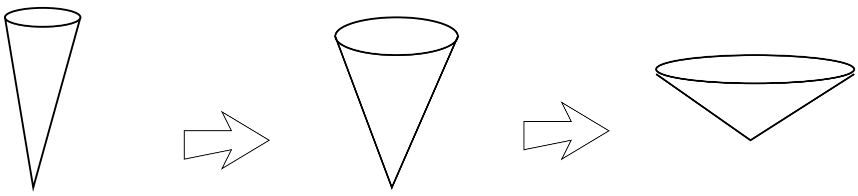

Figure 4: $\mathbb{C}/\mathbb{Z}_N$ orbifolds have always tachyons in the closed string spectrum. By turning on some of this tachyons the cone expands till reaching flat space.

$\mathbb{C}/\mathbb{Z}_{2N+1} \to \mathbb{C}/\mathbb{Z}_{2N-1}$ (see figure 4). Notice that in the orbifold description in [5], the order of the orbifold is odd while in our case is even. However as we have already said, the correspondence between the two systems is expected to happen only at large N.

Notice that in both sides, brane-antibrane annihilation and the vev of the twisted field are discontinuous, so our approximation of continuous mass variation has no meaning between these points. When interpreted in terms of branes and antibranes, we have seen that the radius of the bolt takes discrete values as well as the c parameter. Notice that, as discussed in [5], the process of desingularising the cone is expected to be discontinuous. So one expects sudden changes in the volume of the bolt from both sides. For example, one can consider the emission of dilaton fields by the brane-antibrane annihilation into the bulk. That will correspond to a sudden change in the M-theory coordinate that looks like the cone change in the twisted orbifold side as described in [5]. It will be very interesting to relate these two discontinuous processes in detail. From the M-theory point of view, we can see the bolt as emitting waves that change suddenly the shape of the cone till the bolt disappear to a point.

It is important to notice that we are not mapping open string to closed string tachyons, we are just comparing the behaviour and evolution of two different systems related by an M-theory lift. If one naively tries to map one open to one closed string tachyon, one immediately realises that things are not working. For large number of pairs of branes and antibranes N the counting of open string tachyons goes like N^2 but the number of twisted closed string tachyons grows like N. Also the perturbative masses of these states do not match. However, the number of steps driving the system to the supersymmetric configuration is the same, of order N. This is because when a pair brane-antibrane disappears there are also N open string tachyons that decouple from the spectrum.

5. From orbifolds to branes

The relation among $\mathbb{C}/\mathbb{Z}_N$ orbifolds and D6-$\overline{\text{D6}}$ systems in the large bolt limit leads us to consider a non-supersymmetric orbifold of type IIA, perform its trivial lift to M-theory and reduce it afterwards along a circle inside the orbifold. The configuration thus obtained cannot be trusted far away from the origin, but it must correspond to the local description of some D6-brane system.[4] Notice that this is exactly what happens for the supersymmetric orbifold $\mathbb{C}^2/\mathbb{Z}_{N(\pm 1)}$: this produces the familiar supersymmetric A_{N-1} orbifolds (for review see [20, 21]), as reviewed at the beginning of section 3, which upon reduction along the Hopf fibre, gives rise to the local description of a system of N coincident D6-branes located at the fixed points of the S^1 along which we performed the reduction.

We shall next consider some particular non-supersymmetric orbifold singularities of the form $\mathbb{C}^2/\left(\mathbb{Z}_N \times \mathbb{Z}_M\right)$, where the action of each subgroup is defined in such a way that the complete orbifold breaks supersymmetry completely. We will see that after reduction along the Hopf fibre, the type-IIA configuration has a line of conical singularities with some fractional D6-branes located at the origin whenever $M \neq 0$. It is important to stress, once more, that the forthcoming analysis is only reliable close to where the D-Branes are located.

5.1 $\mathbb{C}^2/(\mathbb{Z}_N \times \mathbb{Z}_M)$ orbifolds

Let us define polar coordinates in $\mathbb{C}^2$ by

$$\begin{aligned} z_1 &= r\cos\frac{\theta}{2}\, e^{i(\psi+\varphi)/2} \\ z_2 &= r\sin\frac{\theta}{2}\, e^{i(\psi-\varphi)/2}\,, \end{aligned}$$

[4]That is similar to what is happenning in flux-branes, see for instance, the discussion on ref [19].

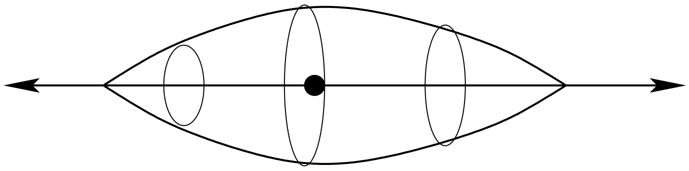

Figure 5: Reduction of a non-supersymmetric $\mathbb{C}^2/(\mathbb{Z}_N \times \mathbb{Z}_M)$ orbifold. At a fix distance from the origin where the fractional D6-brane is located the S^2 presents two conical singularities that represents the intersection of the two dimensional sphere with a line of $\mathbb{C}/\mathbb{Z}_M$ singularities.

where the range of the different angular variables is $0 \leq \theta < \pi$, $0 \leq \varphi < 2\pi$ and $0 \leq \psi < 4\pi$. The action of the $\mathbb{Z}_N \times \mathbb{Z}_M$ group on $\mathbb{C}^2$ is of the form:

$$g_1(z) = \begin{pmatrix} e^{\frac{2\pi i}{N}} & 0 \\ 0 & e^{\frac{2\pi i}{N}} \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \tag{5.1}$$

and

$$g_2(z) = \begin{pmatrix} e^{\frac{2\pi i}{M}} & 0 \\ 0 & e^{\frac{-2\pi i}{M}} \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}, \tag{5.2}$$

where g_i are the generators of the group. The orbifold does not preserve supersymmetry because each subgroup preserves supersymmetries of different chirality. Thus, whenever the order of both subgroups (N, M) is different from 1, the total orbifold breaks supersymmetry completely. Due to the identifications associated with the orbifold construction, there are now two cones associated with each of the subgroups

$$0 \leq \varphi < \frac{2\pi}{M} \qquad \text{and} \qquad 0 \leq \psi < \frac{4\pi}{N}.$$

One can work with angular variables satisfying the standard periodicity conditions by rescaling $\{\varphi, \psi\}$. In this way, the periods are manifest in the metric

$$ds^2 = ds^2(\mathbb{E}^{1,6}) + dr^2 + \frac{r^2}{4}\left[d\theta^2 + sin^2\theta\frac{d\varphi^2}{M^2} + \left(\frac{d\psi}{N} + cos\theta\frac{d\varphi}{M}\right)^2\right]. \tag{5.3}$$

There are many S^1's along which one could reduce, but we shall take the usual Hopf fibering, i.e. reducing on ψ. Using the Kaluza-Klein ansatz, the ten dimensional metric in the string frame looks like

$$ds^2 = \frac{r}{2N}\left\{ds^2(\mathbb{E}^{1,6}) + dr^2 + \frac{r^2}{4}\left(d\theta^2 + (\sin\theta)^2\frac{d\varphi^2}{M^2}\right)\right\}, \tag{5.4}$$

whereas the dilaton and RR one form are given by:

$$\begin{aligned} e^{\Phi} &= \left(\frac{r}{2N}\right)^{3/2} \\ C_{(1)} &= \frac{N}{M}\cos\theta d\varphi\,. \end{aligned} \tag{5.5}$$

Notice that if $M = 1$, the above configuration matches the local description of N coincident D6-branes close to the naked singularity $(r = 0)$, and half of the supersymmetry is preserved.

Whenever $M \neq 1$, the naked singularity remains but there is an additional line of conical singularities coming from a $\mathbb{C}/\mathbb{Z}_M$ orbifold. Indeed, after reducing along the Hopf fibering, we are left with $\mathbb{R}^3$ in the subspace transverse to the D6-branes, but with one angular coordinate of reduced period.[5] The set of fixed points of the orbifold which reduced the period of the angular variable is given by the line $\theta = 0, \pi\ \forall r$. We can thus interpret the ten dimensional configuration as the local description close to $r = 0$ of a set of D-branes on a $\mathbb{C}/\mathbb{Z}_M$ orbifold carrying fractional charge N/M.

Notice that for the systems just discussed there is no open-closed string instability correspondence like in previous sections, since the analysis in both sides implies the existence of closed string tachyons in the twisted sectors.

6. Conclusions and perspectives

In this paper, we have analysed the geometry of the lift to M-theory of certain D6-$\overline{\text{D6}}$ systems. For any non-BPS configuration, we find a bolt type singularity. The annihilation of D6-$\overline{\text{D6}}$ pairs in the open string description is realised, on the gravity side, by a reduction on the size of the bolt and a desingularization of the conical singularities on it. In the large bolt limit, the M-theory geometry is locally described by $\mathbb{C} \times \mathbb{C}/\mathbb{Z}_N$. This allowed us to qualitatively match the annihilation of D6-$\overline{\text{D6}}$ pairs with the sequences of transitions described in $\mathbb{C}/\mathbb{Z}_N$ non-supersymmetric orbifolds. As we have already said, the process is discontinuous in both sides. It would be very interesting to analyse how the discrete evolution is produced. Having realised this connection, we considered the non-supersymmetric orbifold $\mathbb{C}^2/\mathbb{Z}_N \times \mathbb{Z}_M$ and its relation with a local description of unstable branes, which turned out to be fractional D6-branes on a $\mathbb{C}/\mathbb{Z}_M$ singularity.

There are several natural questions related with the results reported here. Due to the relation among D6-brane systems and $\mathbb{C}/\mathbb{Z}_N$ orbifolds, it would be very interesting to investigate if there is any brane realisation for the sequences of transitions found in [5] regarding non-supersymmetric $\mathbb{C}^2/\mathbb{Z}_{N(k)}$ orbifolds.

We would also like to point out that the brane-antibrane system discussed in this paper can be interpreted as a particular case of a pair of D6-branes at generic angles, the one in which they have opposite orientations. These more general systems do generically break supersymmetry[6] and in some regions of their moduli space, they are empty of tachyons. It would be interesting to understand the M-theory dynamics in these cases [23].

Other physical systems which have recently been given a lot of attention and do also have localised closed string tachyons are fluxbranes [24]. It would be interesting to understand the stability and supersymmetry of some of them using similar local descriptions to the ones appearing in this work.

On the other hand, the analysis in section 5 is just a local one, as can be seen from the fact that the dilaton (string coupling) increases as we move away from the origin. It would be nice to look for non-BPS configurations whose validity of description goes beyond the region where the brane sits.

[5] Using $\varphi' = \varphi/M$, the metric is flat but φ' is defined over $0 \leq \varphi' < 2\pi/M$.

[6] See, among others, [22].

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